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Measurements of the SUSY Higgs self-couplings and the reconstruction of the Higgs potential

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Abstract

We address the issue of the reconstruction of the scalar potential of a two-Higgs doublet model having in mind that of the MSSM. We first consider the general CP conserving dim-4 effective potential. To fully reconstruct this potential, we show that even if all the Higgs masses and their couplings to the standard model particles are measured one needs not only to measure certain trilinear Higgs self-couplings but some of the quartic couplings as well. We also advocate expressing the Higgs self couplings in the mass basis. We show explicitly, that in the so-called decoupling limit, the most easily accessible Higgs self-couplings are given in terms of the Higgs mass while all other dependencies on the parameters of the general effective potential are screened. This helps also easily explain how, in the MSSM, the largest radiative corrections which affect these self couplings are reabsorbed by using the corrected Higgs mass. We also extend our analysis to higher order operators in the effective Higgs potential. While the above screening properties do not hold, we argue that these effects must be small and may not be measured considering the foreseen poor experimental precision in the extraction of the SUSY Higgs self-couplings.

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1 Introduction

The most important issue at the upcoming colliders is the elucidation of the mechanism of symmetry breaking and the hunt for the Higgs. Within the Standard Model, $\mathcal{SM}$, there is a strong indirect evidence that the latter might be light. But at the same time within the $\mathcal{SM}$ such a light Higgs poses the problem of naturalness. Supersymmetry, SUSY, solves this problem and for a large array of models predicts a light Higgs in accordance with the present precision data. The task of the next colliders will therefore be not so much the discovery of the (lightest) Higgs but a careful study of the properties of the Higgs system since this will be an ideal window on the mechanism of (super)symmetry breaking. Many state-of-the-art studies have analysed the couplings of the lightest Higgs to fermions and to gauge bosons. Most useful conclusions are in the context of a next linear collider (LC) [1], for a summary see [2, 3]. One can for example discriminate between a light Higgs within the $\mathcal{SM}$ and one within a supersymmetric model. Precision studies on other supersymmetric particles that may be produced at these colliders can nicely complement these studies. However one key ingredient that still requires further studies and simulations concerns the important aspect of the Higgs potential. Not only because this triggers electroweak symmetry breaking but also because supersymmetry breaking is also encoded in this potential. Some studies[4, 5, 6, 7, 8] have addressed the issue of the measurements of some of the Higgs self-couplings within minimal SUSY. These studies have been rather purely phenomenological studies in the sense that one has, within the minimal supersymmetric model (MSSM), quantified various cross sections for double Higgs (and for some triple) Higgs production at a high energy collider. From these one has derived a sensitivity on some individual Higgs self-couplings by varying the strength of these couplings independently, while fixing the Higgs mass spectrum. We would however expect that a deviation in one of the Higgs self-couplings should not only affect other Higgs self-couplings but also affect the Higgs mass spectrum. Moreover, one would like to see how a particular deviation in the Higgs self-couplings relates to the fundamental parameters of the Higgs potential and also what order of magnitude should one expect from these deviations. This can most efficiently be addressed through an effective potential approach and would be similar to what has been applied in the measurements of the trilinear[9] or even quadrilinear[10] self-couplings of the weak vector bosons. Within the one Higgs doublet of the $\mathcal{SM}$ a general parameterisation of the Higgs self-couplings has been given[11, 12] and its effects at the colliders studied[11, 12, 13]. As for the case of SUSY where one needs two Higgs doublets such a study is missing although a leading order parameterisation has been known[14]. The aim of this paper is to fill this gap. As we will see this parameterisation of the scalar potential is important, not only it can embody through an effective potential many of the well known radiative corrections[15, 16, 17, 18, 19] but it will also make clear the link between what can be learned about the Higgs potential by measuring the Higgs masses and what additional, if any, information can be gained if one can study the self-couplings. This will also show that the couplings of the charged Higgs to the neutral Higgses can embody the same information as some of the trilinear neutral Higgs couplings. The measurement of the Higgs self-couplings involving the charged Higgses have, as far as we know, never been

[1] A Higgs factory at a muon facility gives astounding results[1], however many technical problems need to be solved before the design of such a facility.

addressed although a calculation of charged Higgs pair production at the LHC has been made[6, 20]. Our findings also help understand why the radiative corrections to the Higgs self-couplings of the lightest Higgs in the MSSM though substantial (as are those to the Higgs mass) become tiny when expressed in terms of the lightest Higgs mass[21]. With the five $(h, H, A, H^{\pm})$ Higgses of the MSSM one has 8 possible Higgs trilinear self-couplings. One would then think that, together with the measurements of the Higgs masses and their couplings to ordinary matter the measurement of these trilinear self-couplings would allow a full reconstruction of the leading order dim-4 effective Lagrangian describing the Higgs potential. Even in the most optimistic scenarios where all the Higgses are light, we find that a full reconstruction requires the measurements of some quartic couplings which are extremely difficult, if not impossible[22], to measure even at the linear collider. In the decoupling regime[14]where only one of the Higgses is light and with properties very much resembling those of the $\mathcal{SM}$, the effective parameters of the Higgs potential will be screened and thus extremely difficult to measure. One can, of course, entertain that new physics affecting the Higgs potential appears as higher dimensional operators, dim-6, in which case the above "screening" effects are not operative. However one expects these effects to be too small to be measured considering the expected accuracy, no better than $\sim 10\%$, at which the Higgs self-couplings are to be probed (for a nice review see[2, 3, 22]).

2 Leading order parameterisation of the Higgs potential and the Higgs self-couplings

When trying to parameterise the effects of some new physics on the properties of *familiar* particles, the effective Lagrangian is most useful. One uses all the known symmetries of the model and then writes the tower of operators according to their dimensions. One expects thus that the allowed lowest dimension operators have most impact on the low energy observables, higher order operators being suppressed due to the large mass scale needed for their parameterisation. Therefore, for the minimal SUSY the lowest dimension operator is of dim-4. For the MSSM one needs two Higgs doublets, H_1 and H_2 with opposite hypercharge, $Y = \mp 1$ respectively. They may be written as

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(h_1 + i\varphi_1^0) \\ \varphi_1^- \end{pmatrix} \quad H_2 = \begin{pmatrix} \varphi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(h_2 + i\varphi_2^0) \end{pmatrix} \tag{2.1}$$

Requiring $\mathcal{CP}$ conservation one can write following[14],

$$\begin{aligned} V_{eff} \;=\; & (m_1^2 + \mu^2)|H_1|^2 + (m_2^2 + \mu^2)|H_2|^2 - [m_{12}^2(\epsilon H_1 H_2) + h.c.] \\ & + \frac{1}{2}[\frac{1}{4}(g^2 + g'^2) + \lambda_1](|H_1|^2)^2 + \frac{1}{2}[\frac{1}{4}(g^2 + g'^2) + \lambda_2](|H_2|^2)^2 \\ & + [\frac{1}{4}(g^2 - g'^2) + \lambda_3]|H_1|^2|H_2|^2 + [-\frac{1}{2}g^2 + \lambda_4](\epsilon H_1 H_2)(\epsilon H_1^* H_2^*) \\ & + (\frac{\lambda_5}{2}(\epsilon H_1 H_2)^2 + [\lambda_6|H_1|^2 + \lambda_7|H_2|^2](\epsilon H_1 H_2) + h.c.) \end{aligned} \tag{2.2}$$

ϵ is the antisymmetric matrix with $\epsilon_{12} = -1$. g and g' are the SU(2) and U(1) gauge couplings. μ is a supersymmetry preserving mass term. The case with all λ_i being zero corresponds to the original MSSM potential at tree-level. Moreover, exact supersymmetry imposes that $m_1^2 = m_2^2 = m_{12}^2 = 0$ so that no electroweak symmetry breaking ensues. m_1, m_2 and m_{12} are thus essential for electroweak symmetry breaking and encode also supersymmetry breaking. These dimensionful quantities are soft SUSY breaking parameters. As for the λ's, practically all analyses of the Higgs phenomenology have only viewed them as "soft" terms originating from higher orders loop effects. As known these loop effects can be substantial as they are enhanced by large Yukawa couplings (corrections are quartic in the top mass) and have kept the MSSM alive, see [23] for example. However it may well be that models of supersymmetry breaking can provide a *direct* contribution to these parameters, for instance through non-renormalisable operators in the Kähler potential. Technically these contributions would be deemed *hard*. It has however been stressed recently[24, 25] that if these parameters are related to the source of soft SUSY breaking then they would not destabilise the scalar potential and would evade the "un-natural" quadratic divergence problem, thus leading to a viable model. In such circumstances such terms may lead to the lightest SUSY Higgs with a mass much in excess of 150GeV and with "no un-naturalessness" dilemma. Therefore, the reconstruction of the Higgs potential is crucial. In a different context, supersymmetric models with a warped fifth dimension[26], it was shown that some of the quartic couplings (apart from the usual ones of gauge origin) can even be supersymmetric and originate from a non-minimal Kähler potential. Supersymmetry breaking terms also contribute to the latter as well as to the quadratic terms. However to obtain a satisfactory electroweak breaking in this scheme constrains the parameters such that $t_\beta = 1$ is picked up (see below the definition of t_β).
Of course, the approach we follow and the results we obtain can be made to apply directly to a general 2-Higgs doublet model(2HDM) , one only needs to switch the gauge couplings contributions off in the potential Eq. 2.2. This said when we investigate precision measurements extracted from the Higgs bosons to fermions one should stick to a model whose characteristics are close to the MSSM. The 2HDM we have in mind is the so-called type-II, in the terminology of [27], where the down-type quarks and leptons couple to H_1 and the up-type quarks to H_2 as in the MSSM. However in most studies of the 2HDM $\lambda_{6,7}$ are not considered on the basis that they may induce too large FCNC.
This brings us to the issue concerning the order of magnitude for the various λ_i in a general suspersymmetric context. For the conventional MSSM, one effectively gets a Yukawa enhanced contribution, starting at one loop, which affects all seven paramaters[23]. The largest contribution for moderate $\tan\beta$ stems from λ_2 and is of order .1. In [24] where "natural hard" terms are discussed, some orders of magnitude for the λ_i are given based on how and at what scale SUSY breaking is transmitted. Values as high as 1. could be entertained. Such values ($< 4\pi$) are still perturbative. In the warped fifth dimension model[26] it is interesting to note that all λ_i but λ_5 get a SUSY conserving contribution in addition to some SUSY breaking contribution, while λ_5 is of purely SUSY breaking origin. All these contribution disappear in the limit of an infinitely large effective fundamental scale (scale *in lieu* of $M_{\rm Planck}$), which is directly related to the warp factor. The effective couplings can be large enough, in fact so large that the authors estimate a lightest Higgs mass of order 700GeV as a possibility while the couplings are still perturbative up to the cut-off scale. Note however that the cut-off scale is identified with the "low" fundamental scale as compared to the usual Planck scale that is usually used to set the upper bound

on the quartic coupling and hence the mass of the Higgs. Moreover it is argued that the model is safe as regards FCNC [26] .
Some bounds, though not so strong, exist. In all its generality, if we allow some combinations of couplings within the range $-1 < \lambda_i < 1$, independently of $\tan\beta$, can lead to too low values of the Higgs masses (even negative squared masses and problems with vacuum stability can occur). Furthermore to constrain the parameter space some authors have imposed vacuum stability of the potential and perturbativity of the couplings up to high scales[28] as well as tree-level unitarity constraints of the elastic scattering of the Higgses[29] in the 2HDM. Limits from $\Delta\rho$[30] can also be quite useful, although they are model dependent if for instance the stop contribution to $\Delta\rho$ plays a role. Nonetheless all of these requirements still leave a large parameter space that can be quite drastically reduced once the Higgs masses and their couplings are directly measured.

We will therefore follow a general approach based on the potential 2.2, assuming the quadratic terms ($m_{1,2,12}$) to fulfill the usual conditions for a stable minimum with non-vanishing vacuum expectation values. The minimisation of the potential, and the absence of tadpoles, imposes the following constraint on the "soft" SUSY parameters m_1, m_2:

$$\begin{aligned} m_1^2 &= -m_{12}^2 t_\beta - \mu^2 - M_Z^2 c_{2\beta}/2 + v^2(-\lambda_1 c_\beta^2 - \lambda_3 s_\beta^2 - \lambda_4 s_\beta^2 - \lambda_5 s_\beta^2 \\ &+ 3\lambda_6 c_\beta s_\beta + \lambda_7 s_\beta^3/c_\beta) \qquad (2.3) \\ m_2^2 &= -m_{12}^2/t_\beta - \mu^2 + M_Z^2 c_{2\beta}/2 + v^2(-\lambda_2 s_\beta^2 - \lambda_3 c_\beta^2 - \lambda_4 c_\beta^2 - \lambda_5 c_\beta^2 \\ &+ \lambda_6 c_\beta^3/s_\beta + 3\lambda_7 c_\beta s_\beta) \qquad (2.4) \end{aligned}$$

where $v^2 = v_1^2 + v_2^2 = 2M_W^2/g^2$, $t_\beta = tan\,\beta = v_2/v_1$, $s_\beta = \sin\beta$ and so on.

Then, the parameter m_{12}^2 can be fixed if we choose M_A, the pseudo-scalar Higgs mass, as an independent variable:

$$m_{12}^2 = -c_\beta s_\beta(M_A^2 + v^2(2\lambda_5 - \lambda_6/t_\beta - \lambda_7 t_\beta)). \qquad (2.5)$$

2.1 The Higgs masses

At this stage, beside M_A and t_β, there are seven independent parameters. Luckily some of these enter the expressions of the Higgs masses and the couplings to fermions and vector bosons.

The mass of the charged Higgs boson reads as

$$M_{H^\pm}^2 = M_A^2 + M_W^2 - v^2(\lambda_4 - \lambda_5). \qquad (2.6)$$

This already shows that a measurement of $M_{H^\pm}$ and M_A can put a limit on the combination $(\lambda_4 - \lambda_5)$. A dedicated study addressing this particular issue at the LHC to differentiate between the MSSM and a general 2HDM has very recently appeared[31].

The two CP-even Higgs states are determined by the mixing angle α. h and H will denote the $\mathcal{CP}$ even Higgs scalars defined through

$$\begin{aligned}
tan\,2\alpha &= \frac{M_A^2 s_{2\beta} + M_Z^2 s_{2\beta} - 2v^2(s_{2\beta}(\lambda_3+\lambda_4) - 2c_\beta^2\lambda_6 - 2s_\beta^2\lambda_7)}{M_A^2 c_{2\beta} - M_Z^2 c_{2\beta} - 2v^2(\lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_5 c_{2\beta} - (\lambda_6-\lambda_7)s_{2\beta})} &(2.7)\\
M_H^2 &= M_Z^2 c_{\alpha+\beta}^2 + M_A^2 s_{\alpha-\beta}^2\\
&\quad +2v^2(\lambda_1 c_\alpha^2 c_\beta^2 + \lambda_2 s_\alpha^2 s_\beta^2 + 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \lambda_5(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2)\\
&- \; 2s_{\alpha+\beta}(\lambda_6 c_\alpha c_\beta + \lambda_7 s_\alpha s_\beta)) &(2.8)\\
M_h^2 &= M_Z^2 s_{\alpha+\beta}^2 + M_A^2 c_{\alpha-\beta}^2\\
&\quad +2v^2(\lambda_1 s_\alpha^2 c_\beta^2 + \lambda_2 c_\alpha^2 s_\beta^2 - 2(\lambda_3+\lambda_4)c_\alpha c_\beta s_\alpha s_\beta + \lambda_5(c_\alpha^2 c_\beta^2 + s_\alpha^2 s_\beta^2)\\
&+ \; 2c_{\alpha+\beta}(\lambda_6 s_\alpha c_\beta - \lambda_7 c_\alpha s_\beta)) &(2.9)
\end{aligned}$$

With $-\pi/2 \le \alpha \le \pi/2$[2], h (H) defines the lightest (heaviest) $\mathcal{CP}$ even Higgs mass. The decoupling limit[32] is usually defined as $M_A \gg M_Z$, we will extend this to mean $M_A \gg v$ (with the λ_{1-7} never exceeding $\mathcal{O}(1)$). Having the decoupling limit in mind it is very instructive and useful to express the dependence in the mixing angle α through $c_{\beta-\alpha}$ and $s_{\beta-\alpha}$ since these two quantities are a direct measure of the couplings of h (and H) to vector bosons and to fermions. In units of the $\mathcal{SM}$ Higgs couplings, the couplings to vector bosons are

$$g_{hVV,HVV} = s_{\beta-\alpha}, c_{\beta-\alpha}$$

while for instance

$$g_{hb\bar{b}} = -s_\alpha/c_\beta = s_{\beta-\alpha} - t_\beta c_{\beta-\alpha} \qquad (2.10)$$

Therefore, especially if t_β has been identified from other SUSY processes, these combinations could be easily extracted once the light Higgs has been produced at the linear collider, and after allowing for some (important) QCD and in some cases model dependent vertex corrections. The couplings of H can be easily translated from those of h by the substitution $h \to H, M_h \to M_H, s_{\beta-\alpha} \to c_{\beta-\alpha}, c_{\beta-\alpha} \to -s_{\beta-\alpha}$. Moreover $c_{\beta-\alpha}$ is a very good measure of decoupling since in this limit $c_{\beta-\alpha} \sim 1/M_A^2$. To wit,

$$\begin{aligned}
&M_A^2 c_{\beta-\alpha}s_{\beta-\alpha} \sim M_A^2 c_{\beta-\alpha} \to &(2.11)\\
&s_{2\beta}c_{2\beta}\left\{M_Z^2 - v^2\left(\lambda_3+\lambda_4+\lambda_5+(\lambda_6-\lambda_7)t_{2\beta} - \lambda_1\frac{c_\beta^2}{c_{2\beta}} + \lambda_2\frac{s_\beta^2}{c_{2\beta}} - \frac{\lambda_6}{t_\beta} - \lambda_7 t_\beta\right)\right\}
\end{aligned}$$

It is important to keep in mind for later reference that in the decoupling limit we have:

$$c_{\beta-\alpha}\;,\;(M_A^2 - M_H^2) \to \mathcal{O}(1/M_A^2) \qquad (2.12)$$

In the decoupling limit, the lightest Higgs mass writes as

[2] $tan\,2\alpha$, does not fully define α, we should have given both $s_{2\alpha}$ and $c_{2\alpha}$.

$$M_h^2 \to M_Z^2 c_{2\beta}^2 + 2v^2 \left\{ \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + s_{2\beta}((\lambda_3+\lambda_4+\lambda_5)s_\beta c_\beta - (\lambda_6+\lambda_7) - (\lambda_6-\lambda_7)c_{2\beta}) \right\} \tag{2.13}$$

These simple considerations already show that if some of the λ_{1-7} are not too tiny one should observe their effects by measuring the Higgs masses and also the Higgs couplings to ordinary fermions. This MSSM generalisation as pointed out in[24] can allow for a lightest Higgs mass in excess of 150GeV, say, independently of t_β.

2.2 The Higgs self-couplings

Some of the expressions below, for the Higgs trilinear couplings, have been given elsewhere[14, 6]. For completeness we list all the couplings. Introducing

$$g_h = \frac{2M_W}{g} = \sqrt{2}v \tag{2.14}$$

we have

$$\begin{aligned}
g_{hhh} &= 3g_h\lambda_{hhh} \\
\lambda_{hhh} &= -\frac{e^2}{s_{2W}^2} s_{\beta+\alpha}c_{2\alpha} \\
&+ \lambda_1\, c_\beta s_\alpha^3 - \lambda_2\, c_\alpha^3 s_\beta + \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)\, s_{2\alpha}c_{\beta+\alpha} \\
&+ \lambda_6 s_\alpha^2(c_{\beta+\alpha} + 2c_\alpha c_\beta) + \lambda_7 c_\alpha^2(c_{\beta+\alpha} - 2s_\alpha s_\beta) \qquad (2.15) \\
g_{Hhh} &= -3g_h\lambda_{Hhh} \\
\lambda_{Hhh} &= \frac{e^2}{s_{2W}^2}\left(s_{\beta+\alpha}s_{2\alpha} - \frac{1}{3}c_{\beta-\alpha}\right) \\
&+ \lambda_1\, c_\beta s_\alpha^2 c_\alpha + \lambda_2\, s_\beta c_\alpha^2 s_\alpha - \frac{1}{2}(\lambda_3+\lambda_4+\lambda_5)\,(s_{2\alpha}s_{\beta+\alpha} - \frac{2}{3}c_{\beta-\alpha}) \\
&+ \lambda_6 s_\alpha(c_\beta c_{2\alpha} + c_\alpha c_{\beta+\alpha}) - \lambda_7 c_\alpha(s_\beta c_{2\alpha} + s_\alpha c_{\beta+\alpha}) \qquad (2.16) \\
g_{hHH} &= 3g_h\lambda_{hHH} \\
\lambda_{hHH} &= \frac{e^2}{s_{2W}^2}(c_{2\alpha}s_{\beta+\alpha} - \frac{2}{3}s_{\beta-\alpha}) \\
&+ c_\alpha s_\alpha(\lambda_1 c_\alpha c_\beta - \lambda_2\, s_\alpha s_\beta) - \frac{(\lambda_3+\lambda_4+\lambda_5)}{2}\,(s_{\beta+\alpha}c_{2\alpha} - \frac{s_{\beta-\alpha}}{3}) \\
&+ (\lambda_6 c_\alpha(c_\beta c_{2\alpha} - s_\alpha s_{\beta+\alpha}) + \lambda_7 s_\alpha(c_{2\alpha}s_\beta + c_\alpha s_{\beta+\alpha}) \\
g_{hAA} &= g_h\lambda_{hAA} \\
\lambda_{hAA} &= -\frac{e^2}{s_{2W}^2} s_{\beta+\alpha}c_{2\beta} \\
&+ \lambda_1\, c_\beta s_\beta^2 s_\alpha - \lambda_2\, s_\beta c_\beta^2 c_\alpha - (\lambda_3+\lambda_4+\lambda_5)\,(s_\beta^3 c_\alpha - s_\alpha c_\beta^3) + 2\lambda_5 s_{\beta-\alpha}
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \lambda_6 s_\beta (c_{2\beta} s_\alpha + c_\beta s_{\beta+\alpha}) + \lambda_7 c_\beta (c_{2\beta} c_\alpha - s_\beta s_{\beta+\alpha}) && (2.17)\\
g_{hH^+H^-} &= g_h \lambda_{hH^+H^-} \\
\lambda_{hH^+H^-} &= \lambda_{hAA} - \frac{e^2}{2s_W^2} s_{\beta-\alpha} + (\lambda_4 - \lambda_5) s_{\beta-\alpha} && (2.18)\\
g_{HHH} &= -3 g_h \lambda_{HHH} \\
\lambda_{HHH} &= \frac{e^2}{s_{2W}^2} c_{2\alpha} c_{\beta+\alpha} \\
&+ \quad c_\alpha^3 c_\beta \lambda_1 + s_\alpha^3 s_\beta \lambda_2 + \frac{(\lambda_3 + \lambda_4 + \lambda_5)}{2} s_{2\alpha} s_{\beta+\alpha} \\
&- \quad \lambda_6 c_\alpha^2 (s_\beta c_\alpha + 3 s_\alpha c_\beta) - \lambda_7 s_\alpha^2 (c_\beta s_\alpha + 3 c_\alpha s_\beta) \\
g_{HAA} &= -g_h \lambda_{HAA} \\
\lambda_{HAA} &= -\frac{e^2}{s_{2W}^2} c_{\beta+\alpha} c_{2\beta} \\
&+ \quad \lambda_1 \, c_\beta s_\beta^2 c_\alpha + \lambda_2 \, s_\beta c_\beta^2 s_\alpha + (\lambda_3 + \lambda_4 + \lambda_5) \, (s_\beta^3 s_\alpha + c_\beta^3 c_\alpha) - 2\lambda_5 c_{\beta-\alpha} \\
&+ \quad \lambda_6 s_\beta (c_\beta c_{\beta+\alpha} + c_\alpha c_{2\beta}) - \lambda_7 c_\beta (s_\beta c_{\beta+\alpha} + s_\alpha c_{2\beta}) && (2.19)\\
g_{HH^+H^-} &= -g_h \lambda_{HH^+H^-} \\
\lambda_{HH^+H^-} &= \lambda_{HAA} + \frac{e^2}{2s_W^2} c_{\beta-\alpha} + (\lambda_5 - \lambda_4) c_{\beta-\alpha} && (2.20)
\end{aligned}
$$

Written this way the expressions are not very telling, moreover it is clear that some of the couplings must be related since after having measured the masses, there remains only three independent λ while there are 8 Higgs trilinear couplings.

3 Expressing the self-couplings in the mass basis

The writing of the Higgs self-couplings in terms of the fundamental parameters λ_i is not the most judicious. The reason is that all these parameters, though in different combinations, already appear in the expression for the Higgs masses. Therefore the Higgs masses already constrain the Higgs potential, even if partially. Since the Higgs masses (or at least the lightest Higgs) will be measured first, the trilinear Higgs self-couplings being accessed only trough double Higgs production whose cross section is small, one should ask how much more do we learn from the measurement of the trilinear couplings once we have measured the masses. For example, it is quite likely that the heavy Higgses are too heavy so that one only has access to the lightest $\mathcal{CP}$ even Higgs. In this situation one would have an extremely precise determination of its mass (either at the LHC or a linear collider) and also a very precise determination of its couplings to fermions and (gauge bosons) and hence of the angle α (or the combination $\alpha - \beta$) at a linear collider. One should therefore use this information, namely trade these precisely measured physical quantities for two of the parameters λ_i and then re-express the self-coupling $h/H\ hh$, in terms of these physical parameters. Of course the choice of the λ_i is not unique, however the parameterisation of the self-couplings in terms of masses will be more transparent and would have the advantage of including information on some previously measured quantities. We therefore propose to use the physical basis, using as input all the Higgs masses and the mixing angle α[3]. For the latter, beside t_β, one can use the quantities $s_{\beta-\alpha}$ (and $c_{\beta-\alpha}$) which can be directly extracted from the Higgs couplings to fermions or the vector bosons. For instance a good measurement of the production cross section of h at e^+e^- furnishes $s_{\beta-\alpha}$. The angle β may be measured in some purely non-Higgs processes or if one has access to some heavy Higgses also through a study of their couplings to matter [4]. Thus apart from M_A, one can trade $M_h, M_H, M_{H^\pm}, \alpha$ for, for example, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ through

$$\begin{aligned}
\lambda_4 &= (M_A^2 + M_W^2 - M_{H^\pm}^2)/v^2 + \lambda_5 && (3.1)\\
\lambda_1 &= \frac{M_h^2 s_\alpha^2 + M_H^2 c_\alpha^2 - M_Z^2 c_\beta^2 - M_A^2 s_\beta^2}{2v_1^2} - \lambda_5 t_\beta^2 + 2\lambda_6 t_\beta && (3.2)\\
\lambda_2 &= \frac{M_h^2 c_\alpha^2 + M_H^2 s_\alpha^2 - M_Z^2 s_\beta^2 - M_A^2 c_\beta^2}{2v_2^2} - \lambda_5/t_\beta^2 + 2\lambda_7/t_\beta && (3.3)\\
\lambda_3 &= \frac{(M_H^2 - M_h^2) c_\alpha s_\alpha + (M_A^2 + M_Z^2) c_\beta s_\beta}{2v_1 v_2} - \lambda_4 + \lambda_6/t_\beta + \lambda_7 t_\beta. && (3.4)
\end{aligned}$$

With this choice the different Higgs self couplings are given in terms of the Higgs masses and the remaining $\lambda_{5,6,7}$. This choice of basis seems to be the most natural, since the structures that affect λ_{1-4} appear already at "tree-level" and thus should be the most affected by radiative corrections. This is so in the MSSM where the largest corrections at moderate t_β is in λ_2, see [23] for example. Let us first define the combinations,

$$\begin{aligned}
\lambda_a &= \lambda_5 s_{2\beta} - \frac{\lambda_6 + \lambda_7}{2} - \frac{\lambda_6 - \lambda_7}{2} c_{2\beta} \ \ \text{(note that } m_{12}^2 = -M_A^2 s_\beta c_\beta - v^2 \lambda_a)\\
\lambda_b &= \lambda_5 c_{2\beta} + \frac{\lambda_6 - \lambda_7}{2} s_{2\beta} && (3.5)
\end{aligned}$$

After some algebra, where we try as much as possible to express the α dependence through $\alpha - \beta$, we find

$$\begin{aligned}
\lambda_{hhh} &= -\frac{1}{2v^2} s_{\beta-\alpha}\, M_h^2\\
&+ \frac{c_{\beta-\alpha}^2}{s_\beta c_\beta}\left(s_{\beta-\alpha}\lambda_a + c_{\beta-\alpha}\lambda_b + \frac{(M_A^2 - M_h^2)}{2v^2}(s_{2\beta} s_{\beta-\alpha} + c_{2\beta} c_{\beta-\alpha})\right) && (3.6)
\end{aligned}$$

This shows that in the decoupling limit the hhh coupling is completely determined from the measurement of the Higgs mass, M_h and the coupling to WWh/ZZh, $s_{\beta-\alpha}$, both of which should be determined quite precisely from $e^+e^- \to Zh$ at the LC. The λ_a and more so λ_b are totally screened. The result in the decoupling limit is no surprise as it reproduces exactly the $\mathcal{SM}$ result. In this regime one essentially have only one Higgs doublet and if one restricts oneself to dim-4 operators, one reproduces the $\mathcal{SM}$ exactly.

[3] In another context and in a more restricted 2 Higgs doublet model, using the "corrected" masses to express the self-couplings has been advocated in [33, 29].

[4] Of course some of these couplings receive genuine vertex corrections that are *not* encoded in the correction to the angle α, these genuine vertex flavour dependent corrections should be, whenever possible, subtracted before one attempts to extract $s_{\beta-\alpha}$ for example.

$$\begin{aligned}
\lambda_{Hhh} &= c_{\beta-\alpha}\Big\{\frac{4\lambda_a}{3s_{2\beta}}(1-\frac{3}{2}c^2_{\beta-\alpha})+\frac{2\lambda_b}{s_{2\beta}}c_{\beta-\alpha}s_{\beta-\alpha} \;+\; \frac{1}{6v^2}\Big((3M_H^2-2M_h^2) \\
&+ \;\; 4(M_A^2-M_H^2)+\frac{c_{\beta-\alpha}(s_{2\beta}c_{\beta-\alpha}-c_{2\beta}s_{\beta-\alpha})}{s_\beta c_\beta}(2M_h^2+M_H^2-3M_A^2)\Big)\Big\}
\end{aligned} \tag{3.7}$$

In the decoupling limit

$$\lambda_{Hhh}=\frac{c_{\beta-\alpha}}{2v^2}M_H^2 \tag{3.8}$$

Since this coupling could most easily be extracted from $H\rightarrow hh$, M_H would have (together with $c_{\beta-\alpha}$) already been measured, thus the new physics effects are again screened.

The remaining couplings involve at least two heavy Higgses an thus would be more difficult to measure. Nonetheless let us express them in terms of masses. All the couplings will be written as

$$g_{H_iH_jH_k}=\frac{g_h}{s_{2\beta}}\lambda'_{H_iH_jH_k} \tag{3.9}$$

with

$$\lambda'_{hAA} = 2s_{\beta-\alpha}\lambda_a+2c_{\beta-\alpha}\lambda_b \;+\; \frac{1}{2v^2}\Big(c_{\beta-\alpha}c_{2\beta}(2M_A^2-2M_h^2)-M_h^2s_{2\beta}s_{\beta-\alpha}\Big) \tag{3.10}$$

$$\begin{aligned}
\lambda'_{hHH} &= s_{\beta-\alpha}\Big\{\frac{1}{2v^2}\Big[\Big(M_h^2+2(M_H^2-M_A^2)\Big)s_{2\alpha}-2M_A^2c_{\beta-\alpha}s_{\beta+\alpha}\Big] \\
&+ \;\; 2(\lambda_a(1-3c^2_{\beta-\alpha})+3c_{\beta-\alpha}s_{\beta-\alpha}\lambda_b)\Big\}
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\lambda'_{hH^+H^-} &= -2\Big\{\frac{1}{2v^2}\Big[(c_\alpha c_\beta^3-s_\alpha s_\beta^3)M_h^2-(M_A^2-M^2_{H^\pm})c_{\beta+\alpha}-M^2_{H^\pm}c_{2\beta}c_{\beta-\alpha}\Big] \\
&-(s_{\beta-\alpha}\lambda_a+c_{\beta-\alpha}\lambda_b)\}
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\lambda'_{HHH} &= -6\Big\{\frac{1}{2v^2}\Big[s_{\beta+\alpha}\Big((M_H^2-M_A^2)+c^2_{\beta-\alpha}M_A^2\Big)-s_\alpha c_\alpha c_{\beta-\alpha}M_H^2\Big] \\
&+s^2_{\beta-\alpha}(s_{\beta-\alpha}\lambda_b-c_{\beta-\alpha}\lambda_a)\Big\}
\end{aligned} \tag{3.13}$$

Note once more that these couplings, like other H couplings to fermions, sfermions and gauge bosons, can be derived from those of h. For example we do verify that $g_{HHH}=g_{hhh}(h\rightarrow H, M_h\rightarrow M_H, s_{\beta-\alpha}\rightarrow c_{\beta-\alpha}, c_{\beta-\alpha}\rightarrow -s_{\beta-\alpha})$.

$$\begin{aligned}
\lambda'_{HAA} &= -2\Big\{\frac{1}{2v^2}\Big[(s_\alpha c_\beta^3+c_\alpha s_\beta^3)(M_H^2-M_A^2)+c_{\beta-\alpha}c_\beta s_\beta M_A^2\Big] \\
&+s_{\beta-\alpha}\lambda_b-c_{\beta-\alpha}\lambda_a\}
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
\lambda'_{HH^+H^-} &= -2\Big\{\frac{1}{2v^2}\Big[s_{\beta+\alpha}(M_H^2-M_A^2)+c_{\beta-\alpha}c_\beta s_\beta(2M^2_{H^\pm}-M_H^2)\Big] \\
&+s_{\beta-\alpha}\lambda_b-c_{\beta-\alpha}\lambda_a\}
\end{aligned} \tag{3.15}$$

In the physical basis and especially by explicitly displaying the dependence in the mixing angle α through the combination $c_{\beta-\alpha}, s_{\beta-\alpha}$ shows that for couplings involving at least two heavy Higgses

- The dependence in the parameters λ_i only appear through the combination λ_a *or* λ_b. Thus one combination is not accessed in the trilinear couplings
- In the decoupling limit ($M_h \ll M_A, M_H, M_A$), it is only λ_a which is accessible in the self-couplings involving two heavy Higgses (hAA, hHH and hH^+H^-), while in the self-couplings with heavy Higgses only (HHH, HAA, HH^+H^-) only λ_b may be accessed.

Note that the heavy Higgs masses, $M_H, M_A, M_{H^\pm}$, enter the formulae for the couplings only through combinations involving mass differences or $M_A^2 c_{\beta-\alpha}$ so that in effect the self-couplings do not grow with the heavy Higgs mass as occurs in the $\mathcal{SM}$.

3.1 Quartic Higgs couplings

To access the remaining combination we need to consider the quartic couplings. We will write the quartic couplings directly in the physical basis.

Defining

$$\lambda_c = 2c_{2\beta}\lambda_b - \lambda_5 = \lambda_5 c_{4\beta} + \frac{\lambda_6 - \lambda_7}{2} s_{4\beta} \tag{3.16}$$

$$\begin{aligned} g_{hhhh} &= -\frac{3M_h^2}{2v^2}(1 - c^2_{\beta-\alpha}) \\ &- \frac{3}{2v^2 s_\beta^2 c_\beta^2} c^2_{\beta-\alpha}\left\{c_{\beta+\alpha}(s_{\beta-\alpha}s_{2\beta} + c_{\beta+\alpha}c^2_{\beta-\alpha})M_h^2 + s_\alpha^2 c_\alpha^2 M_H^2 - c^2_{\beta+\alpha}M_A^2\right\} \\ &+ \frac{3}{s_\beta^2 c_\beta^2} c^2_{\beta-\alpha}\left\{s_{2\beta}\lambda_a + 2s_{2\beta}c_{\beta-\alpha}s_{\beta-\alpha}\lambda_b + c^2_{\beta-\alpha}\lambda_c\right\} \end{aligned} \tag{3.17}$$

We see again that in the decoupling region one is not sensitive to any of the extra couplings, as expected since we recover the $\mathcal{SM}$ result with only the dim-4 operator. Let us now give the formulae for some of the other quartic couplings to show that some of the novel couplings are not screened in all of the quartic couplings. We will only show the dependence in the extra parameters and will not give the full dependence in terms of masses as otherwise the formulae may be too lengthy.

$$\lambda_{hhhH} \;\rightarrow\; c_{\beta-\alpha}\left\{s_{\beta-\alpha}s_{2\beta}\lambda_a + c_{\beta-\alpha}(3s_{2\beta}\lambda_b + 2c_{\beta-\alpha}s_{\beta-\alpha}\lambda_c - 4c^2_{\beta-\alpha}s_{2\beta}\lambda_b)\right\} \tag{3.18}$$

Again, all the anomalous couplings are screened.

$$\lambda_{hhHH} \;\rightarrow\; s_{2\beta}\lambda_a + 6s_{\beta-\alpha}c_{\beta-\alpha}\left(s_{2\beta}\lambda_b + c_{\beta-\alpha}s_{\beta-\alpha}\lambda_c - 2c^2_{\beta-\alpha}s_{2\beta}\lambda_b\right) \tag{3.19}$$

In $hhHH$ only λ_a may be accessible.

$$\lambda_{hHHH} \quad \rightarrow \quad s_{\beta-\alpha}\left\{s_{\beta-\alpha}s_{2\beta}\lambda_b + c_{\beta-\alpha}\left(s_{2\beta}\lambda_a + 2s^2_{\beta-\alpha}\lambda_c - 4c_{\beta-\alpha}s_{\beta-\alpha}s_{2\beta}\lambda_b\right)\right\} \tag{3.20}$$

In $hHHH$ only λ_b may be accessible.

$$\lambda_{HHHH} \quad \rightarrow \quad s^2_{\beta-\alpha}\left(s^2_{\beta-\alpha}\lambda_c + s_{2\beta}\lambda_a - 2c_{\beta-\alpha}s_{\beta-\alpha}s_{2\beta}\lambda_b\right) \tag{3.21}$$

In $HHHH$ both λ_a and λ_c may be accessible but not λ_b.

3.2 Using another parameterisation

The screening property is a general result which does not depend on which independent parameters we keep beside the physical masses. Had we used another set of independent parameters beside the masses, the same phenomenon would have occurred and only two independent combinations out of the three parameters would enter the expression of the trilinear couplings. Indeed with $\lambda_3, \lambda_5, \lambda_6$ as extra parameters, the role of $\lambda_{a,b,c}$ is played by $\lambda'_{a,b,c}$, such that

$$\begin{aligned}
\lambda_a \rightarrow \lambda'_a &= -s_\beta c_\beta(\lambda_3 - \lambda_5) \\
\lambda_b \rightarrow \lambda'_b &= -\left(\frac{\lambda_3+\lambda_5}{2} + c_{2\beta}\frac{\lambda_3-\lambda_5}{2}\right) + \frac{c_\beta}{s_\beta}\lambda_6 \\
\lambda_c \rightarrow \lambda'_c &= 2\lambda'_b c_{2\beta} - \lambda_5
\end{aligned} \tag{3.22}$$

For instance,

$$\begin{aligned}
\lambda_{hhh} = & -\frac{1}{2v^2}s_{\beta-\alpha}\,M_h^2 \;+\; c^2_{\beta-\alpha}\frac{1}{c_\beta s_\beta}\left(s_{\beta-\alpha}\lambda'_a + c_{\beta-\alpha}\lambda'_b\right) \\
& + \frac{c^2_{\beta-\alpha}}{2v^2 c_\beta s^2_\beta}\Big\{s_{\beta-\alpha}c_\beta s^2_\beta(M_A^2 - M_H^2 - M_h^2 - M_Z^2 c_{2W} + 2M^2_{H^\pm}) \\
& \quad -c_{\beta-\alpha}\Big[s_\beta\left(s^2_\beta(M_A^2 - M_H^2) + c^2_\beta(2M_H^2 + M_Z^2 c_{2W} - 2M^2_{H^\pm} - M_h^2)\right) \\
& \quad +c_{\beta-\alpha}(M_H^2 - M_h^2)\left(s_{\beta-\alpha}c_\beta(1-4s^2_\beta) - c_{\beta-\alpha}s_\beta(3-4s^2_\beta)\right)\Big]\Big\}
\end{aligned} \tag{3.23}$$

Compared to the previous parameterisation, this looks rather more complicated as it involves the charged Higgs mass as well as the heavy H beside the pseudoscalar Higgs mass. Nonetheless all these masses are screened.

4 Dim-4 operators and independent parameters

To easily understand our finding about the screening and the number of independent parameters in the trilinear and quadrilinear couplings, note that the trilinear and quadrilinear couplings originate from the quartic terms λ_i only, Eq. 2.2, whereas the mass terms get an additional contribution from the bi-linear terms m_1, m_2, m_{12} in Eq. 2.2. Take for instance the case of the neutral couplings. The quartic self-couplings emerge as combinations of 5 independent terms in the original fields (before diagonalisation) of the form

$$h_1^4 \ , \ h_2^4 \ , \ h_1^3 h_2 \ , \ h_1^2 h_2^2 \ , \ h_1 h_2^3 \tag{4.1}$$

In terms of the physical scalar fields, h, H:

$$h_1 = -h s_\alpha + H c_\alpha \quad h_2 = h c_\alpha + H s_\alpha \tag{4.2}$$

which we more judiciously write as

$$\begin{aligned} h_1 &= c_\beta(s_{\beta-\alpha} h + c_{\beta-\alpha} H) + s_\beta(H s_{\beta-\alpha} - h c_{\beta-\alpha}) = c_\beta h_- + s_\beta h_+ \\ h_2 &= s_\beta h_- - c_\beta h_+ \end{aligned} \tag{4.3}$$

However keep in mind that h_1 and h_2 in Eq. 2.1 always appear in the form

$$\begin{aligned} v_1 + \frac{h_1}{\sqrt{2}} &= v_1 \left((1 + \frac{h_-}{v'}) + t_\beta \frac{h_+}{v'} \right) \\ v_2 + \frac{h_2}{\sqrt{2}} &= v_2 \left((1 + \frac{h_-}{v'}) - t_\beta^{-1} \frac{h_+}{v'} \right) \end{aligned} \tag{4.4}$$

with $v' = \sqrt{2} v (= g_h$ Eq. 2.14), which helps write the doublets, Eq. 2.1, as

$$\begin{aligned} H_1 &= v_1 \begin{pmatrix} 1 + ((h_- - iG^0) + t_\beta(h_+ + iA))/v' \\ (-G^- + t_\beta H^-)/v \end{pmatrix} \\ H_2 &= v_2 \begin{pmatrix} (G^+ + t_\beta^{-1} H^+)/v \\ 1 + ((h_- + iG^0) - t_\beta^{-1}(h_+ - iA))/v' \end{pmatrix} \end{aligned} \tag{4.5}$$

where $G^{\pm,0}$ stand for the Goldstone bosons.

Therefore in effect the quartic terms originate from a combination of the form

$$(1 + \frac{h_-}{v'})^2 \left(q_1 (1 + \frac{h_-}{v'})^2 + q_2 (1 + \frac{h_-}{v'}) \frac{h_+}{v'} + q_3 \frac{h_+^2}{v'^2} \right) + q_4 (1 + \frac{h_-}{v'}) \frac{h_+^3}{v'^3} + q_5 \frac{h_+^4}{v'^4} \tag{4.6}$$

While the bi-linear terms are of the form

$$b_1 (1 + \frac{h_-}{v'})^2 \ + \ b_2 (1 + \frac{h_-}{v'}) \frac{h_+}{v'} \ + \ b_3 \frac{h_+^2}{v'^2} \tag{4.7}$$

Imposing that no tadpole remains (no linear term in h_+, h_-) means that (b_1, q_1) and (b_2, q_2) must combine such that one has

$$4q_1\frac{h_-^2}{v'^2}(1+\frac{h_-}{v'}+\frac{h_-^2}{4v'^2})+2q_2\frac{h_+h_-}{v'^2}(1+\frac{3h_-}{2v'}+\frac{h_-^2}{2v'^2})$$
$$+\frac{h_+^2}{v'^2}(b_3+q_3(1+\frac{h_-}{v'})^2)+q_4(1+\frac{h_-}{v'})\frac{h_+^3}{v'^3}+q_5\frac{h_+^4}{v'^4} \tag{4.8}$$

Since the coefficients of $h_-^2(q_1)$ and $h_-h_+(q_2)$ are expressed in terms of masses so do those of h_-^3, h_-^4 (q_1) as well as $h_+h_-^2, h_+h_-^3$ (q_2). This does not apply to the trilinear and quadri-linear terms involving h_+^2 and higher order in h_+. Thus for the trilinear terms there are only the two structures $h_+^2h_-$ (with coefficient q_3) and h_+^3 (with coefficient q_4) that can not be expressed solely in terms of the physical masses. The last parameter q_5 only appears in the quartic couplings in the form h_+^4. To obtain q_1 and q_2 is a straightforward matter. One only has to rewrite the $\mathcal{CP}$ -even Higgs masses in terms of $h_\pm$:

$$M_h^2h^2+M_H^2H^2\rightarrow$$
$$h_-^2(M_h^2s_{\beta-\alpha}^2+M_H^2c_{\beta-\alpha}^2)\;+\;2h_+h_-s_{\beta-\alpha}c_{\beta-\alpha}(M_H^2-M_h^2)+h_+^2(M_H^2s_{\beta-\alpha}^2+M_h^2c_{\beta-\alpha}^2) \tag{4.9}$$

q_3, q_4 and q_5 are directly related to the parameters $\lambda_a, \lambda_b, \lambda_c$. The construct of Eq. 4.8 shows that in fact once we get the quartic couplings one also derives the coefficients of the various trilinear couplings. For example, take the quartic couplings as they appear in the original potential in terms of the fields h_1 and h_2,

$$Q_h=\frac{\lambda_1}{2}h_1^4+\frac{\lambda_2}{2}h_2^4+(\lambda_3+\lambda_4+\lambda_5)h_1^2h_2^2-2\lambda_6h_1^3h_2-2\lambda_7h_2^3h_1 \tag{4.10}$$

When expressed in terms of h_- and h_+ and after moving to the mass basis with for example $\lambda_{5,6,7}$ as extra parameters we immediately get the following dependence of the various quartic and trilinear terms

$$(h_+^2+A^2+2H^+H^-)\Big[2(\lambda_ah_--\lambda_bh_+)+\frac{1}{v'}\Big(\lambda_a(h_-^2+G^{0^2}+2G^+G^-)$$
$$-2\lambda_b(h_-h_+-AG^0-(H^+G^-+H^-G^+))+\tilde{\lambda}_c(h_+^2+A^2+2H^+H^-)\Big)\Big] \tag{4.11}$$

with [5]

$$\tilde{\lambda}_c=\frac{c_{2\beta}^2}{s_{2\beta}}\lambda_5-\frac{\lambda_6+\lambda_7}{2}+\frac{\lambda_6-\lambda_7}{2}c_{2\beta}\text{ and }\tilde{\lambda}_c-\lambda_a=\frac{1}{s_{2\beta}}\lambda_c \tag{4.12}$$

In fact with this parameterisation one has that $q_3=\lambda_a$, $q_4=\lambda_b$ and $q_5=\tilde{\lambda}_c$. One can move to another parameterisation, *i.e* choosing a different set of extra parameters, through Eqs. 3.1-3.4. We see that by *completing* the h_-, h_+ dependence of $H_{1,2}$ we even get the full trilinear and quadri-linear couplings involving the pseudo-scalar, charged and Goldstone

[5]The reason λ_c appears instead of $\tilde{\lambda}_c$ depends on how we organise the decoupling and is due to the rewriting of the terms in $s_{\beta-\alpha}^2$ as $1-c_{\beta-\alpha}^2$.

Boson couplings. The completion is obtained by identifying the different t_β dependencies (namely $1, t_\beta, t_\beta^2$) in the modulus of H_1 for instance. It is not hard to see that by re-expressing h_- and h_+ in terms of h and H, we recover all our results. Moreover this writing immediately shows that trilinear scalar couplings involving Goldstone Bosons can all be expressed most simply in terms of the physical Higgs masses only. The requirement for the absence of tadpoles is a crucial one and explains most of our findings when restricting ourselves to dim-4 operators.

5 Radiative Corrections

Our results can also be exploited for easily expressing the radiative corrections to the trilinear (and for that matter quadrilinear) couplings of the SUSY Higgses and explain some of the properties pointed out in the literature. Three-point one-loop radiative corrections for the neutral Higgs system in the MSSM have been calculated [18] within the effective potential approximation[6]. The diagrammatic one-loop radiative corrections to both the trilinear, λ_{hhh} and quadrilinear λ_{hhhh} lightest Higgs self couplings have been revisited recently in[21]. For the case of no mixing in the stop sector it is shown analytically[21] that the bulk of the corrections in the couplings are absorbed by using the corrected Higgs mass while the same is demonstrated numerically for the case of large mixing. One-loop radiative corrections for λ_{hAA} are also considered in [15, 16], for λ_{HAA} in [15] and for λ_{Hhh} in [17]. Here also the corrections are found to be large before re-expressing the results in terms of the corrected masses.

Ref. [23] gives analytical approximations (including 2-loop leading-log corrections) for the effective quartic couplings λ_i[7], using a renormalisation-group improved leading-log approximation. We can adapt their formulae to the one-loop case with large stop masses so that we can compare with the direct calculation of the vertices performed in [18, 5]. For instance, we find the leading one-loop contributions in the limit where the SUSY breaking term, M_S^2, defined below, is (much) larger than the top mass, (which is consistent with our approach of keeping only the dimension-4 operators)

$$\begin{aligned}
\Delta\lambda_{hhh} &= -\frac{g^4 m_t^4}{64\pi^2 M_W^4}\frac{c_\alpha^3}{s_\beta^3}\left\{6\log(M_S^2/m_t^2)\ +3\frac{f_t(c_t+f_t)}{M_S^2}-\frac{c_t f_t^3}{2M_S^4}\right\} \\
\Delta\lambda_{Hhh} &= \frac{g^4 m_t^4}{64\pi^2 M_W^4}\frac{s_\alpha c_\alpha^2}{s_\beta^3}\left\{6\log(M_S^2/m_t^2)\ +\frac{c_t(e_t+2f_t)+f_t(f_t+2e_t)}{M_S^2}-\frac{c_t e_t f_t^2}{2M_S^4}\right\} \\
c_t &= A_t+\mu/t_\beta \quad e_t = A_t+\mu/t_\alpha \quad f_t = A_t-\mu t_\alpha \quad m^2_{\tilde t_1,\tilde t_2} = M_S^2 \pm m_t c_t
\end{aligned} \tag{5.1}$$

These shifts correct the tree-level expression of Eq. 2.15 and Eq. 2.16 respectively. In

[6] See also [5] where expressions are given for some couplings assuming equal soft masses for the stop masses. Note however that there is a misprint in Eq. 2.4 of [5] where in the last term of that equation one should read $A(A+\mu\cot\beta)$ instead of $\mu(A+\mu\cot\beta)$.

[7] Compared to our notations we should make $\lambda_{6,7}\to-\lambda_{6,7}$ in the expressions of [23]. Moreover our sign convention for μ is the opposite of [23] but the same as in [5]. See [34] for a full definition of our conventions.

the limit $\mu \to 0$, all the λ_i in the MSSM vanish but λ_2:

$$\begin{aligned}
\lambda_2^{\tilde{t}_1,\tilde{t}_2} &= \frac{3}{32\pi^2}\frac{g^4 m_t^4}{s_\beta^4 M_W^4}\left\{\log(M_S^2/m_t^2) + \frac{A_t^2}{M_S^2}(1 - A_t^2/12M_S^4)\right\} \\
&\sim .15 \text{ for } A_t = M_S = 1TeV \text{ and } t_\beta = 10
\end{aligned} \tag{5.2}$$

while in the same approximation as Eq. 5.1

$$\begin{aligned}
M_h^2 &= M_Z^2 s_{\beta+\alpha}^2 + M_A^2 c_{\beta-\alpha}^2 + \Delta M_h^2 \\
\Delta M_h^2 &= \frac{3}{8\pi^2}\frac{g^2 m_t^4}{M_W^2}\left(\log(M_S^2/m_t^2) + \frac{f_t c_t}{M_S^2} - \frac{c_t^2 f_t^2}{12 M_S^4}\right)\left(1 + \frac{c_{\beta-\alpha}}{s_\beta^2}(s_{2\beta}s_{\beta-\alpha} + c_{2\beta}c_{\beta-\alpha})\right)
\end{aligned} \tag{5.3}$$

so that one recovers the decoupling property and the fact that the bulk of the radiative corrections are reabsorbed by using the corrected Higgs mass,

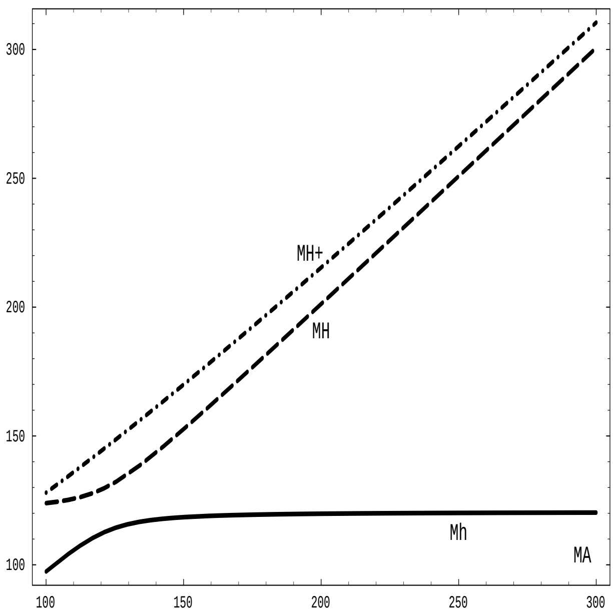
$$\Delta\lambda_{hhh} = -\frac{\Delta M_h^2}{2v^2}\left(s_{\beta-\alpha} + \frac{c_{\beta-\alpha}}{t_\beta}\right) + \frac{3m_t^4}{16\pi^2 v^4}\frac{\mu f_t}{M_S^2}\left(1 - \frac{f_t c_t}{6M_S^2}\right)\frac{c_\alpha^2}{s_\beta^4}c_{\beta-\alpha} \tag{5.4}$$

In a phenomenological analysis of the extraction of the Higgs self-couplings, one could add the contribution of the stop-sbottom at the two-loop level through effective couplings λ_i from the renormalisation group improved results of [23] to which one could include new physics contributions to the λ_i.

Note that contrary to what we have presented in the previous sections, we have shown the "corrections" to the Higgs self-couplings due to radiative corrections (or presence of λ terms) as shifts compared to the tree-level MSSM. We have done so in order to compare with the existing literature[18, 5, 21] that takes into account effects at one-loop only. Although this shows that the bulk of the corrections is absorbed in terms of the Higgs mass, the notion of shifts here is somehow misleading especially that some of the correction is contained in the "corrected" mixing angle α.

6 Phenomenology and reconstruction of the Higgs potential at future colliders

As we have seen, the measurement of the entire set of the dim-4 operators which is necessary to reconstruct the Higgs potential in SUSY (and 2HDM) requires that one crosses the thresholds for the production of 3 Higgs bosons, which is not an easy task especially that the cross sections will get tinier and tinier as the Higgs multiplicity increases. As we have seen also, a precise measurement of the Higgs masses and their couplings to ordinary matter is an important ingredient in the reconstruction of this potential. The LHC can thus give a first hint on the parameters λ_i. For instance imagine that the LHC discovers some SUSY particles and identifies them as such but that one discovers also that the lightest Higgs has a mass in excess of 150GeV. This would point to a scalar potential with "hard" λ terms. We could probably even set a rough bound on their possible values. A LC with enough energy to produce some of the Higgses and good luminosity to probe their couplings would constitute a nice complementary machine though. Although double Higgs production at the LHC[7, 8, 20, 35, 6] may not be so negligible, extracting the trilinear Higgs self-couplings will prove a challenge[8]. Therefore for the rest of this session we will only briefly outline what might be measured from the self-couplings of the Higgs at different stages of the LC. However, before doing so, let us illustrate what the mass measurements alone can bring and how the spectrum can be drastically affected by different forms of the potential. As an illustration we stick to $\tan\beta = 10$ and consider the situation where the λ receive corrections from the stop sector with the following parameters $A_t = 1000$GeV, $M_S = 800$GeV,$\mu = -300$GeV. We will compare the situation where no "hard" terms are added with a situation with $\lambda_{1-5} = -\lambda_{6,7} = .1$, *i.e.* of the order of $\lambda_2^{\tilde{t}_1\tilde{t}_2}$. The mass spectrum of the Higgs system for this choice of parameters is shown in Fig. 1



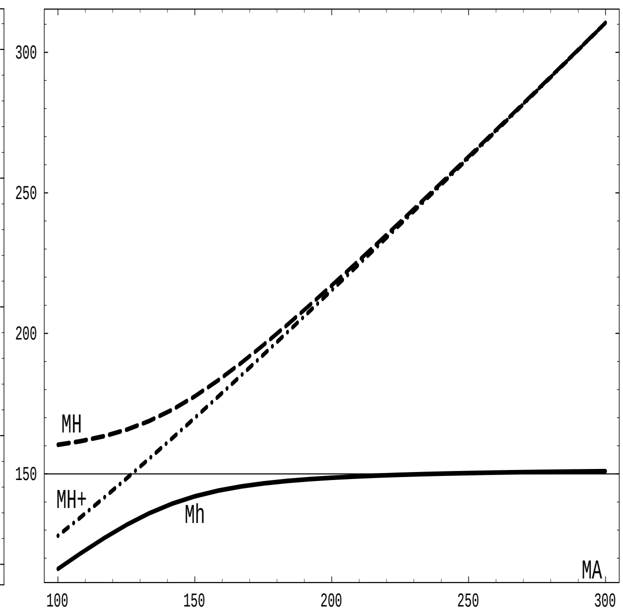


Figure 1: *Higgs mass spectrum without "hard" terms but with $A_t = 1000GeV$, $M_S = 800GeV$, $\mu = -300GeV$ and $\tan\beta = 10$ (left) and with the inclusion of additional terms with $\lambda_{1-5}^{\rm new} = -\lambda_{6,7}^{\rm new} = .1$(right). All masses are in GeV.*

One striking feature is that M_h can be substantially heavier than what it is in the usual MSSM, while the mass ordering between $M_{H^\pm}$ and M_H is certainly another distinguishing feature for this particular choice of parameters.

The rate of h production at e^+e^- , weighted by $s^2_{\beta-\alpha}$, can also provide a helpful hint and additional constraint. However, decoupling although slightly delayed by the presence of the new λ_i, occurs rather fast in this variable as shown in Fig. 2. Having measured $\tan\beta$ greatly helps as the figure illustrates.

A full analysis from the measurements of the masses and the couplings to fermions and vector bosons is left to a forthcoming detailed analysis[36].

As for the Higgs self-couplings the presence of λ_i can have a drastic effect as shown in Fig. 3 especially for small M_A, see in particular the swing in g_{HHH}. In this region

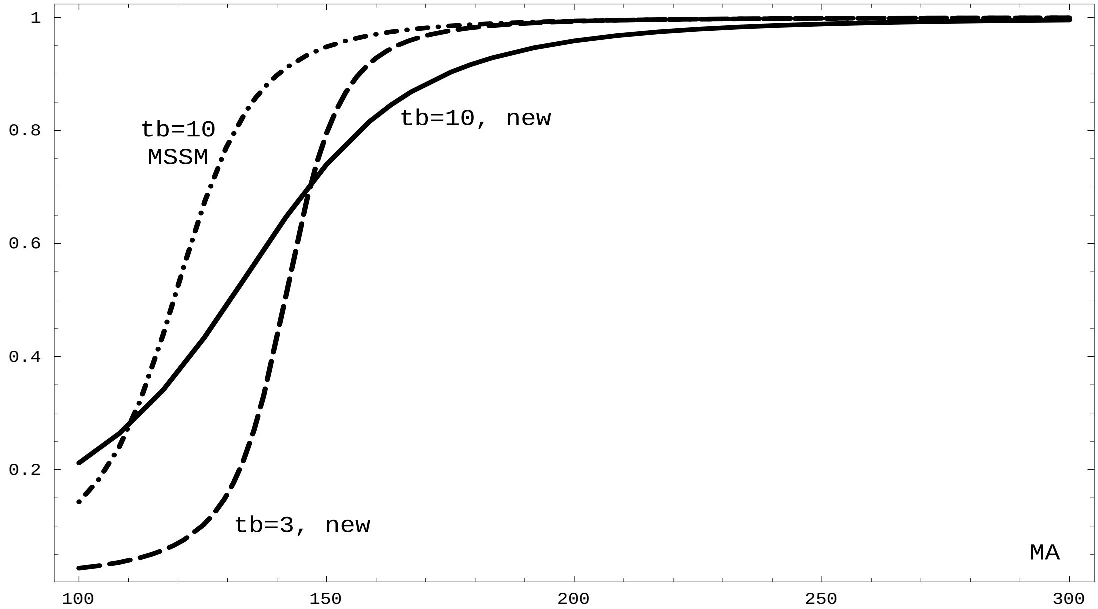


Figure 2: $s^2_{\beta-\alpha}$ *as a function of* M_A*. MSSM refers to* $\lambda_i = 0$ *with* $A_t = 1000 GeV$*,* $M_S = 800 GeV$*,*$\mu = -300 GeV$ *while "new" has* $\lambda^{\rm new}_{1-5} = -\lambda^{\rm new}_{6,7} = .1$*. tb stands for* $\tan\beta$*.*

one expects though that all Higgs masses would have been measured and thus a good constraint on the parameter space will have been provided by the masses. As soon as we enter the decoupling region, the largest coupling is g_{hhh} which reaches its SM value. The other couplings remain unfortunately rather small although some have values larger than their corresponding $\mathcal{SM}$ value. However as we have seen the bulk of these deviations is due to the rather large deviations in the Higgs masses. In this respect let us note that Fig. 3 seems to indicate that the HHH coupling can get rather large for small M_A. However observe that we have plotted a reduced coupling in units of the SM coupling $\tilde{g}^{\rm SM}_{hhh} = -3M_h^2/2v^2$. The reason the reduced coupling attains a value larger than 1 is due to the larger mass of H and that we are in a region of non decoupling, see Fig. 2. In this region H is more standard-like than h, as far as its couplings to gauge bosons are concerned. Had we used $\hat{g}^{\rm SM}_{hhh} = -3M_H^2/2v^2$ as a unit, the reduced coupling would be below 1.

Let us now review briefly how an e^+e^- machine working at successive thresholds for Higgs production can attempt to unravel the Higgs potential.

6.1 Stage 1

Imagine a situation where no heavy Higgs has been produced at a first stage of a linear collider at 500GeV or the LHC, we would then be in the decoupling limit. The only trilinear couplings which may be accessed are hhh and Hhh through $e^+e^- \rightarrow Zhh$ (fusion channels are not efficient at these energies and Higgs masses). However there is no sensitivity to Hhh. Indeed the amplitude for $e^+e^- \rightarrow Zhh$, in the unitary gauge, can be written as

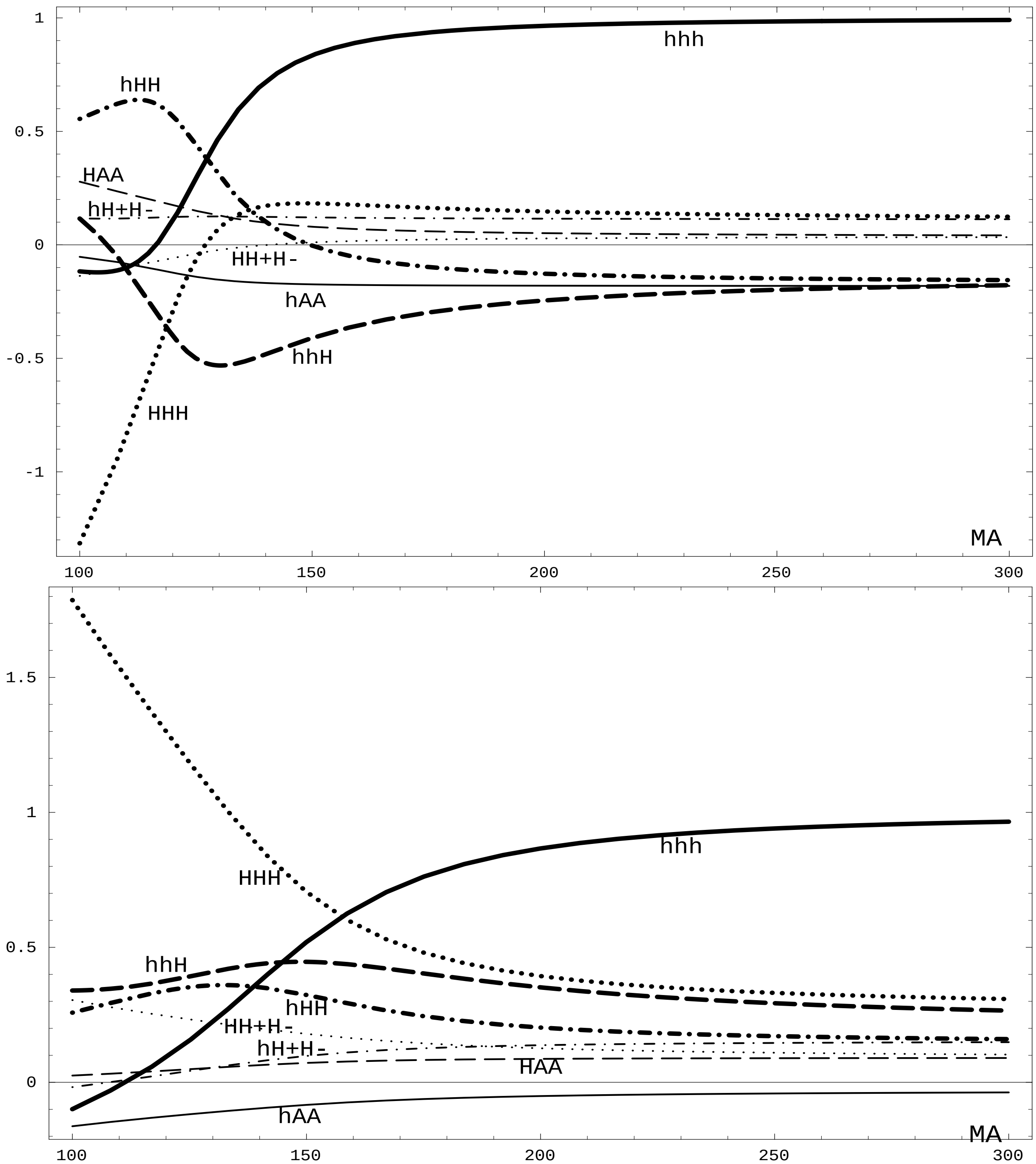


Figure 3: $g_{H^iH^jH^k}/\tilde{g}^{\rm SM}_{hhh}$ *with* $\tilde{g}^{\rm SM}_{hhh} = -3M_h^2/2v^2$. *The SM Higgs mass is identified with* M_h *(and thus varies with* M_A*). The curve in the first panel is the usual MSSM where the one in the second panel is defined with the same parameters as in Fig. 1.*

$$\mathcal{M}_{Zhh} = a_h \lambda_{hhh} s_{\beta-\alpha} \; + \; a_H \lambda_{Hhh} c_{\beta-\alpha} + R_a \tag{6.1}$$

where R stands for other contributions not containing the trilinear Higgs couplings. We have seen that λ_{Hhh} is screened by a factor $c_{\beta-\alpha}$ (Eq. 3.7), it is further screened by another such factor when we consider its contribution to this cross section.

At this stage the best would be to reconstruct as precisely as possible M_h and the couplings of h to fermions and the vector bosons. This will help give a bound on the λ_i. If one makes some model dependent assumptions on the λ_i(imposing some discrete or global symmetries), this can be used to extract some information on M_A. If an independent measurement of $\tan\beta$ is missing at the time of the measurements of the Higgs properties, this will complicate the analyses.
If, on the other hand, the mixing angle is such that $c_{\beta-\alpha}$ is not too small and that M_A is not too large, $e^+e^- \;\; \to ZH$ may be accessible. Then the coupling λ_{Hhh} could be reached directly through $H \to hh$. This may still turn out not to be too helpful, since we have seen that the λ_i are still screened in this coupling, even though the screening in this situation could be mild. Moreover H decays into other particles ($t\bar{t}$ or $b\bar{b}$,..) and superparticles (charginos and neutralinos) may still be dominant so that $Br(H \to hh)$ will be poorly determined. Let us remark at this point that most of the nice analyses of the SUSY Higgs self-couplings[3, 22] that have been performed were done solely in the context of the minimal supersymmetric model, with no additional "hard" terms in the potential, and have relied heavily on the extremely good precision of the measurement of the dominant branching ratio into $b\bar{b}$. In case h is heavier than 150GeV these analyses need to be extended.

6.2 Stage 2

For a machine with higher energies where H and A and thus most probably $H^\pm$ have been discovered, the first thresholds for double Higgs production (after that of Zhh) may be

$$\begin{aligned} e^+e^- \;\; &\to ZhH, \nu_e\bar{\nu}_e Hh \text{ with } \; \mathcal{M}_{Hh} = b_h \lambda_{Hhh} s_{\beta-\alpha} \; + \; b_H \lambda_{HHh} c_{\beta-\alpha} + R_b \\ e^+e^- \;\; &\to hhA \text{ with } \; \mathcal{M}_{Ahh} = c_h \lambda_{hhh} c_{\beta-\alpha} \; + \; c_H \lambda_{Hhh} s_{\beta-\alpha} + c_A \lambda_{hAA} c_{\beta-\alpha} + R_c \end{aligned} \tag{6.2}$$

Again, unfortunately these two reaction will not be very sensitive to deviations in the trilinear couplings if one takes into account the screening effect in λ_{Hhh}. Fusion processes could also be exploited at this stage and the next, but they exhibit a similar behaviour to the annihilation processes as far the extraction of the parameters is concerned.

6.3 Stage 3

With higher energies one produces two heavy Higgses in association with a light Higgs or a Z.

$$
\begin{aligned}
e^+e^- &\to ZHH, \nu_e\bar{\nu}_e HH \text{ with } \mathcal{M}_{HH} = d_h\lambda_{HHh}s_{\beta-\alpha} \;+\; d_H\lambda_{HHH}c_{\beta-\alpha} + R_d \\
e^+e^- &\to ZAA, \nu_e\bar{\nu}_e AA \text{ with } \mathcal{M}_{ZAA} = e_h\lambda_{hAA}s_{\beta-\alpha} \;+\; e_H\lambda_{HAA}s_{\beta-\alpha} + R_e \\
e^+e^- &\to hHA \text{ with } \mathcal{M}_{AHh} = (f_h\lambda_{HHh} \;+\; f_H\lambda_{hAA})s_{\beta-\alpha} + (f'_h\lambda_{Hhh} + f'_H\lambda_{HAA})c_{\beta-\alpha} + R_f \\
e^+e^- &\to H^+H^-h \text{ with } \mathcal{M}_{H^+H^-h} = g_h\lambda_{H^+H^-h} + R_g \\
e^+e^- &\to ZH^+H^-, \nu_e\bar{\nu}_e H^+H^- \text{ with } \mathcal{M}_{XH^+H^-} = h_h\lambda_{hH^+H^-}s_{\beta-\alpha} \;+\; h_H\lambda_{H^+H^-}c_{\beta-\alpha} + R_h
\end{aligned}
\tag{6.3}
$$

As can be seen all of these reactions will be used to determine λ_a (λ_b will still be screened). Let us give some idea about the order of magnitude of the cross sections to show that things can get really tough. As a reference take all extra contributions to the λ_i to be vanishing with SUSY parameters as those considered in the introduction of this section: $A_t = 1000$GeV, $M_S = 800$GeV, $\mu = -300$GeV and $\tan\beta = 10$ and take $M_A = 300$GeV. The third stage could be taken as $\sqrt{s} = 1.2$TeV. We find that ZHH and ZAA are about 2.3 10^{-2}fb, while the other processes listed in this stage are 2 orders of magnitude below. Before taking into account signatures and efficiencies this can amount to about only 25 events a year based on a luminosity of 1ab^{-1}.

6.4 Stage 4

At even higher energies, production of three heavy Higgs bosons could in principle allow to determine λ_b. The processes at our disposal will be

$$
\begin{aligned}
e^+e^- &\to AAA && \text{with } \mathcal{M}_{AAA} = i_h\lambda_{hAA}c_{\beta-\alpha} \;+\; i_H\lambda_{HAA}s_{\beta-\alpha} \\
e^+e^- &\to HHA && \text{with } \mathcal{M}_{HHA} = j_h\lambda_{hHH}c_{\beta-\alpha} \;+\; j_H\lambda_{HHH}s_{\beta-\alpha} + j_A\lambda_{HAA}s_{\beta-\alpha} + R_j \\
e^+e^- &\to H^+H^-H && \text{with } \mathcal{M}_{H^+H^-H} = k_H\lambda_{H^+H^-H} + R_k \\
e^+e^- &\to H^+H^-A && \text{with } \mathcal{M}_{H^+H^-A} = l_h c_{\beta-\alpha}\lambda_{H^+H^-h} + l_H s_{\beta-\alpha}\lambda_{H^+H^-H}
\end{aligned}
\tag{6.4}
$$

Cross sections here are very small here. For instance for the set of parameters considered above and with $\sqrt{s} = 2$TeV, AAA production is about 1.410^{-7}fb! Although a full study allowing a much larger parameter range (including M_A) is in order, it seems that a λ_b measurement would be out of reach.

6.5 Stage 5

As we have seen earlier (see Eq. 4.11) the effect of the third combination of parameters, λ_c, can only be observed in processes involving a vertex with 4 Higgses. The first threshold where such a vertex contributes is a $Zhhh$ final state, which we could have classified in stage 2 (with a ZHh final state). However even for a $\mathcal{SM}$ Higgs $Zhhh$ or $\nu_e\bar{\nu}_e hhh$ at a 10TeV LC with a luminosity as high as 10^{35}cm^{-2}s^{-1} yields only about 5 events per year[22]! Thus the prospect for a useful measurement looks grim especially that in $hhhh$ the λ_c effect is screened as $c^4_{\beta-\alpha}$! Quartic couplings where this contribution is not screened involve any combination of the heavy Higgses ($H^{\pm}, H, A$). Triple H production $ZHHH$ is not operative since it is triggered by ZH production while quadruple production of the heavy Higgses are too tiny to be exploited. Thus a full reconstruction may prove to be impossible if the full set λ_{1-7} is present.

7 Effects from higher order operators

Up to now we have only discussed the effects of the dim-4 operators. Higher order operators are doomed to contribute less significantly, as their effects are explicitly screened by a high scale. We will illustrate this case by considering only three new operators, and restrict ourselves to a few Higgs self couplings to make the point. We consider

$$V_{eff} \to V_{eff} + \frac{1}{\Lambda^2}\left\{\tilde{\kappa}_1(H_1H_1^*)^3 + \tilde{\kappa}_2(H_2H_2^*)^3 + \tilde{\kappa}_3(H_1H_1^*)^2(H_2H_2^*)\right\} \quad (7.1)$$

For notational ease we will use

$$\kappa_i = \frac{v^2}{\Lambda^2}\tilde{\kappa}_i \quad (7.2)$$

with Λ the scale of new physics. We find

$$\begin{aligned} \lambda^{\kappa}_{hhh} &= -\frac{e^2}{s^2_{2W}} s_{\beta+\alpha}c_{2\alpha} \\ &+ (\lambda_1 - 6\kappa_1 c^2_\beta - 2\kappa_3 s^2_\beta)\, c_\beta s^3_\alpha - (\lambda_2 - 6\kappa_2 s^2_\beta)\, c^3_\alpha s_\beta + \frac{1}{2}((\lambda_3+\lambda_4-2\kappa_3 c^2_\beta)+\lambda_5)\, s_{2\alpha}c_{\beta+\alpha} \\ &+ \lambda_6 s^2_\alpha(c_{\beta+\alpha}+2c_\alpha c_\beta) + \lambda_7 c^2_\alpha(c_{\beta+\alpha}-2s_\alpha s_\beta) \\ &- 4\kappa_1 c^3_\beta s^3_\alpha + 4\kappa_2 s^3_\beta c^3_\alpha + 4 s_\beta c^2_\beta c_\alpha s^2_\alpha \kappa_3 \end{aligned} \quad (7.3)$$

Note that we have split the effect of the new contributions in two parts. The first (second line of Eq. 7.3) can be viewed as a shift in $\lambda_{1,2,3}$ while the other (last line in Eq. 7.3) can be considered as a genuine new contribution beyond the effects of the dim-4 operators. The shifts mean that the combinations $(\lambda_1 - 6\kappa_1 c^2_\beta - 2\kappa_3 s^2_\beta)$,$(\lambda_2 - 6\kappa_2 s^2_\beta)$ and $(\lambda_3 - 2\kappa_3 c^2_\beta)$ replace $\lambda_{1,2,3}$, respectively, in the definition of α, $m_{h,H}$ in Eqs. 2.7-2.9. Again, this means that even in the absence of any dim-4 operator, the dim-6 operators as defined above will also affect the Higgs masses and couplings to fermions and vector bosons. Moving to the mass basis, keeping as extra parameters $\lambda_{5,6,7}$ and $\kappa_{1,2,3}$ we get

$$\lambda^{\kappa}_{hhh} = \lambda_{hhh} \; - 4\kappa_1 c^3_\beta s^3_\alpha + 4\kappa_2 s^3_\beta c^3_\alpha + 4 s_\beta c^2_\beta c_\alpha s^2_\alpha \kappa_3 \quad (7.4)$$

and

$$\lambda^{\kappa}_{Hhh} = \lambda_{Hhh} - 4\left(c^3_\beta s^2_\alpha c_\alpha \kappa_1 + s^3_\beta c^2_\alpha s_\alpha \kappa_2 - (\frac{2}{3} - s^2_\alpha)c^2_\beta s_\beta s_\alpha \kappa_3\right) \quad (7.5)$$

Where $\lambda_{H/h\,hh}$ are given in Eqs. 3.6-3.7.

We see that the higher order operators are not further reduced by the decoupling factor $c_{\beta-\alpha}$ and that all κ_i contribute to all the self-couplings, unlike with λ_i where we are only left with a combination of two couplings. This means that if one ideally have measured all the masses and couplings to ordinary fermions and quite precisely all the trilinear Higgs self-couplings one could tell whether higher order operators are contributing. However considering the foreseen precision on the extraction of the Higgs self-couplings and the expected small contribution of the higher order terms, this would seem to be overly optimistic.

8 Conclusion

A dedicated study of double Higgs production at a high luminosity LC[3, 22] within the $\mathcal{SM}$ has shown that it is very difficult to extract the Higgs self-couplings with a precision better than 20% in the first stage of a LC improving to slightly better than 10%[22] at a multi-TeV LC facility, even in the most favourable case of a Higgs light enough that decays into $b\bar{b}$. In the decoupling limit, the lightest SUSY Higgs will have properties very similar to that of the $\mathcal{SM}$ and thus we would also get a precision on its self-coupling with a very similar precision. Unfortunately we have shown that in this limit once we have measured the mass of the Higgs (which will be known at better than the per-mil level) and its couplings to the ordinary $\mathcal{SM}$ particles (with a precision of a few per-cent), the precision attained in the self-couplings will not be sufficient to reveal new physics. Indeed effects from "anomalous" operators affecting the Higgs potential have a direct impact on the Higgs mass and the couplings to fermions. When these are taken into account additional effects in the self-couplings are screened either by mixing angles (dim-4 operators) or large scales. Even if we are not in the decoupling regime and even if we restrict oneself to the leading dim-4 operators, we have shown that measurements of all the possible trilinear self-couplings would not allow to reconstruct the most general lowest dimension Higgs potential. To achieve this one needs to measure some of the quartic couplings. However an analysis that would take into account the measurements of the Higgs masses and their couplings to the ordinary particles should give some useful constraints. An analysis along these lines completed with the extraction of some of the trilinear self-couplings at different stages of the LC is under way[36].

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